High-energy behavior of cross sections in theories with large extra dimensions

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We discuss the high-energy behavior of cross sections in theories with large extra dimensions and low-scale quantum gravity, addressing two particular issues: (i) the tension of the D-branes, and (ii) bounds on the cross section and their relation to approximations in the mode sum over Kaluza-Klein-graviton exchanges.

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Theories with large compact dimensions and an associated low scale characterizing quantum gravity have been the subject of intensive study recently (a few references are given in [1,2]). In Ref. [2] we analyzed some of the phenomenological aspects of such theories, including the high-energy behavior of cross sections for reactions involving graviton exchange. Assuming that there are n large compact dimensions of size $\sim r_n$, and denoting the scale of quantum gravity in the $(4+n)$-dimensional space as $M_{4+n} \gtrsim M_s$ (where M_s is the string mass scale in an underlying string theory), one finds the relation

$$M_{Pl}^2 = (r_n)^n (M_{4+n})^{2+n} \tag{1}$$

where $M_{Pl} = G_N^{-1/2} = 1.2 \times 10^{19}$ GeV is the Planck mass, obtained from the measured Newton constant G_N. Thus, in these theories, the largeness of M_{Pl} is seen to be a consequence of the largeness of the compact dimensions, and the underlying short-distance Planck mass, M_{4+n} can actually be much less than M_{Pl}. At large distances $r \gg r_n$ (here for simplicity we assume the same compactification radius for all of the n compact dimensions), the gravitational force $F \propto r^{-2}$, while for $r \ll r_n$, this changes to $F \propto r^{-(2+n)}$. It is a striking fact that there is a vast extrapolation of 31 orders of magnitude between the smallest scale of 0.2 mm to which Newton's law has been tested [3] and the scale that has conventionally been regarded as being characteristic of quantum gravity, namely the Planck length, $L_{Pl} = 1/M_{Pl} \sim 10^{-33}$ cm, and it is quite possible that new phenomena could occur in these 31 decades that would significantly modify the nature of gravity. It is therefore instructive to explore how drastically one can change the conventional scenario in which both gauge and gravitational interactions occur in four-dimensional spacetime up to energies comparable to the Planck mass. String or brane theory involves extra dimensions and, especially in type I constructions, naturally lead to models in which open strings end on D_p branes [4], so that the standard-model gauge and matter fields propagate on these branes, while closed strings (gravitons) propagate in the ''bulk'' between the branes.

In these types of models, gravitational scattering is strongly altered by the presence of a large number of Kaluza-Klein (KK) modes for the graviton. To an observer in the usual four-dimensional space, the massless graviton is replaced by a set of KK modes, of which the lowest is the massless graviton itself, but the others are massive. The mass of a KK graviton mode is

$$\mu_{l_1,\ldots,l_n} = \left(\sum_{i=1}^{n} l_i^2 \right)^{1/2} r_n^{-1} \tag{2}$$

where the mode numbers are $l_j \in \mathbb{Z}$ and the indices $j = 1,2,\ldots,n$ run over the number of extra compactified dimensions. The couplings of all of these KK states to other particles have the same Lorentz structure as the coupling of the graviton. These KK-graviton modes have the effect of changing the strength of gravitational scattering from its naive value $\sim G_N = M_{Pl}^{-2}$ to a level not much less than regular weak strength, if the underlying quantum gravity scale M_s is not much greater than the electroweak scale, $M_{ew} \sim 250$ GeV. This can be seen because in calculating the rate for such a scattering process, one multiplies the squared coupling M_{Pl}^{-2} by a factor incorporating the multiplicity of the various KK modes that are exchanged. Since this factor is $\sim (s^{1/2} r_n)^n$, where $s^{1/2}$ is the center-of-mass energy, when one substitutes the expression for r_n from Eq. (1), the factor of $1/M_{Pl}^2$ is exactly cancelled, and the final product is $s^{n/2}/M_{4+n}^{n+2}$. Thus, from a four-dimensional viewpoint, although the KK gravitons are coupled extremely weakly, this is compensated by their very large multiplicity, so that their net effect involves in the denominator a mass scale of order M_{4+n} [6].

In Ref. [2] we carried out estimates of the effects of the exchange of the KK-graviton modes to $2 \to 2$ gravitational scattering processes. In the theories of interest here, as $\sqrt{s}$ becomes comparable to the string scale, M_s, one changes over from a field theory (with effects of D-branes included) to a fully stringlike picture, so that M_s serves as an upper cutoff to the low-energy effective field theory in which the calculation is performed. Accordingly, one imposes an upper cutoff

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$$l_i < l_{max} = M_s r_n \tag{3}$$

on the sums over KK modes, which thus run over the range

$$l_i = 0, \pm 1, \ldots, \pm l_{max} \quad \text{for} \quad i = 1, \ldots, n. \tag{4}$$

The value of $l_{max} = M_s r_n$ is very large; for example, for $n = 2$, for $M_s \sim 30$ TeV, one has $l_{max} = M_s r_2 \sim 4 \times 10^{14}$.

In [2] we studied several approaches to estimating the high-energy behavior of the 2-2 gravitational scattering cross section and chose an eikonal approximation that automatically produced a unitary result for this cross section. Our result was [see Eq. (2.35) in Ref. [2]]

$$\sigma(s) = \frac{4\pi s}{M_{4+n}^4} \tag{5}$$

where s is the center of mass energy squared. It was emphasized that because of the universal nature of the gravitational coupling, this applies to any 2-2 scattering process, independent of particle type.

One application of this result is to ultrahigh energy (UHE) neutrinos with energies ranging from $\sim 10^{15}$ eV to 10^{20} eV and beyond. Neutrinos in these energy regions can be produced in several ways, including (i) from active galactic nuclei, (ii) as decay products of the pions produced by the reaction $p + \gamma_{CMB} \to N + \pi$, where γ_{CMB} denotes a photon in the cosmic microwave background radiation, (iii) via analogous reactions in which the protons scatter off of the radiation field of a source such as a gamma-ray burster, and (iv) via decays of Z bosons produced resonantly from antineutrinos scattering on relic neutrinos [7]. From our estimates, in conjunction with standard-model calculations of ultra-high energy νp and $\bar{\nu} p$ total scattering cross sections [8], we concluded that in theories with large compactification radii and low-scale quantum gravity, it is possible for gravitational scattering to make a non-negligible contribution to these cross sections. This could also affect the opacity of the Earth to ultra-high energy neutrinos. Indeed, from standard-model calculations, it is known that at energies $E \gtrsim 10^{15}$ eV, the interaction length for (anti)neutrinos is smaller than the diameter of the Earth, and for $E \gtrsim 10^{18}$ eV, the Earth is opaque to (anti)neutrinos [8], so that the highest-energy (anti)neutrinos would be expected to arrive at a detector from the upward hemisphere, while those coming upward and thus traversing more of the earth are commensurately more highly absorbed en route. If, indeed, new gravitational contributions to ultra-high energy νp and $\bar{\nu} p$ scattering are significant, this would be important for currently operating large detectors such as AMANDA and BAIKAL, and the next-generation detectors such as AUGER, NESTOR, ANTARES, ICECUBE, etc. [7].

It is of continuing interest to investigate further what predictions an assumed underlying string theory of quantum gravity could make concerning models having extra dimensions whose compactification radii are large. Unfortunately, at the present stage of understanding, one cannot derive the compactification process *ab initio* or reliably calculate the compactification radii. Another property is the tension of the D_p branes [4] that serve as the p-dimensional spaces, forming $(p+1)$-dimensional world volumes, in which the standard-model fields propagate, while gravitons propagate in the higher-dimensional "bulk" extending outside of these D_p branes. This tension (more precisely, energy per unit p-dimensional spatial volume of the D-brane) τ_p is given by [5]

$$\tau_p = \frac{1}{g_s (2\pi)^p (\alpha')^{(p+1)/2}} \tag{6}$$

where $\alpha' = M_s^{-2}$ is the Regge slope, which sets the scale of the string mass M_s and is related to the string tension T according to $T = 1/(2\pi\alpha')$. Because of the inverse dependence on the string coupling g_s, the D-brane tension, where one can calculate it reliably using perturbative string theory (i.e., for small g_s), satisfies $\tau_p \gg (M_s)^{p+1}$. More generally, since nonperturbative effects can be important, one expects that τ_p is at least of order $(M_s)^{p+1}$. In turn, this determines the rigidity of the D-branes; for large tension, recoil effects accompanying KK-graviton emission are negligibly small. It follows that KK-graviton recoil effects are negligible up to the energy $E \sim M_s$ at which the low energy pointlike effective field theory methods used in phenomenological studies [1,2] cease to apply and must be supplanted by full string-theoretic calculations. Consequently, in most analyses [1,2], recoil effects are usually dropped. Indeed, even if one takes a moderately strong string coupling $g_s \sim O(1)$, τ_p would still be of order $(M_s)^{p+1}$ (as is clear from simple model-independent dimensional arguments). In the exceptional case where one assumes that $\tau_p \ll (M_s)^{p+1}$, the recoil effects would suppress KK-graviton emission and hence various cross sections involving these [9,10].

Our eikonal approximation used for calculating the KK-graviton 2-2 scattering cross section automatically yields a unitary result, and, indeed, this is one reason that we relied upon this method for our estimate [2]. As we noted in [2], the Froissart bound does not apply to KK-graviton scattering, because this bound assumes that the lightest particle exchanged in the t-channel has a finite mass, but this is not the case for reactions involving KK-graviton exchange, since the lowest-mass particle is the usual massless graviton. Indeed, if one lets m_{min} be the lowest-mass particle exchanged in the t-channel and s_0 be a reference value for the center of mass energy squared, then the Froissart bound is [11,12]

$$\sigma < \frac{4\pi}{m_{min}^2} \ln^2(s/s_0). \tag{7}$$

Thus, clearly, there is no bound if $m_{min} = 0$, as is the case here.

Since the enhanced size of the cross sections involving KK-graviton exchange comes from the large multiplicity of KK modes, one might inquire what sort of result one would get by making an approximation in which one excludes the single massless graviton from the sum over KK modes. Recall that these modes are indexed by the n additional KK momenta $k_i = l_i / r_n$ where $l_i \in \mathbb{Z}$ and the graviton itself has

$l_i=0$ for $i=1,\ldots,n$. In this case there is a finite, though very small, gap, $m_{min}=1/r_n$. In [2] we concentrated on the case $n=2$, so let us take this again here for definiteness; in this case, $r_2=M_{Pl}/M_6^2$. The Froissart bound then allows (neglecting the logs) $\sigma=4\pi/m_{min}^2=4\pi r_2^2$. This is exactly the value of our linearly rising cross section (5) at $s=M_{Pl}^2$. In this case there are just four KK-graviton exchanges that are characterized by the minimum nonzero mass, namely, for $n=2$, $k_\lambda=(\pm 1/r_2,0)$, $(0,\pm 1/r_2)$. The strength of the partial wave amplitude s/M_{Pl}^2 of each exchange is order unity at this value $s=M_{Pl}^2$, thereby allowing for the saturation (up to logs) of the Froissart bound. For $s>M_{Pl}^2$, the cross section saturates at $4\pi r_2^2$, as is clear from the fact that larger values would require larger impact parameters $b>r_2$, but here the spacetime is four-dimensional and no special KK-graviton effects occur.

Consider next some lower energy $s=M_{Pl}^2/N^2$. The relevant KK modes are the $O(N^2)$ modes satisfying the condition $k_1^2+k_2^2<(N/r_2)^2$, i.e. the modes that have squared masses smaller than $(N/r_n)^2$. In this case the Froissart bound allows cross sections $\sigma=4\pi r_2^2/N^2$, again in agreement with our Eq. (5). Furthermore the reduction of each individual exchange amplitude by the factor $s/M_{Pl}^2=1/N^2$ is compensated by the multiplicity of the relevant $\sim N^2$ KK-graviton exchanges, so that again we have a total resultant amplitude of order unity and hence saturation of the Froissart bound [6]. Of course for the true cross section, one cannot exclude the contribution of the massless graviton, which, as noted, means that the conventional Froissart bound does not apply to this cross section. In passing, we note that in Ref. [10] an attempt to analyze the high-energy behavior due to KK graviton was made, and seemed to give a somewhat weaker growth of the cross section than in Eq. (5). We have traced the reason for this; instead of the t-dependence $1/(t-\mu_{l_1,\ldots l_n}^2)$ appropriate for the exchange of the KK-graviton mode with extra-dimensional momenta $(l_1/r_n,\ldots,l_n/r_n)$, Ref. [10] used an *ad hoc* exponential form. As the derivation of the Froissart bound clearly shows, it is the t-dependence (equivalently, the dependence on the impact parameter, b) that matters and not the initial s^ν behavior of the Born amplitude that, even for $\nu\geqslant 2$, is removed by unitarization in systems with finite range interactions.

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